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13. ABSTRACT (Maximum 200 words)  The goal of this work is to develop robust schemes for the simulation of crack propagation using adaptive finite element analysis. Here we address the main problem of estimation of the error in the engineering-quantity of interest (e.g., stress-intensity factors for the crack, the stress in the "hot-spots"). We showed that the error in the quantity of interest can be estimated by employing the standard error estimators, for the energy norm of the error, which are existing in several commercial finite element programs. We have developed a computer-based approach for checking the quality of the error estimators used in practice and we have also developed error estimators which are robust for meshes used in engineering practice. This work has led to an innovative approach for adaptive mesh-refinement to obtain the quantity of interest with prescribed tolerance.					
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**ADAPTIVE h-p FINITE ELEMENT METHODS FOR THE NUMERICAL  
SIMULATION OF CRACK PROPAGATION**

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**U.S. ARMY RESEARCH OFFICE  
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#### I. Scientific Progress and accomplishments:

The major accomplishments in this work can be put in three categories :

1. A computer-based approach for checking the quality of the error estimators,
2. A practical method for reliable estimation of the error in the quantity of interest,
3. An innovative approach for adaptive mesh-refinement to obtain the desired quantity of interest with prescribed accuracy.

The theoretical foundations for the above accomplishments and the significance of these methods in practical computations are given in [1-22]. The above accomplishments provide a strong basis towards developing robust algorithms for the simulation of the crack propagation using adaptive *hp* finite element analysis.

In [1-3] we have shown that the error in the finite element solution at a point can be split into two parts – the *local error* and the *pollution error*. In [3-14] we have shown that standard error indicators which are employed in practice estimate only the local error and hence they will severely underestimate the error in the finite element solution in certain cases. We have developed a new method for the *a-posteriori estimation of the pollution error* by using the standard error indicators for the solution of an auxiliary problem. This method can be implemented in any commercial finite element program which has the capability to solve multiple loads and provide an estimate of the global energy norm of the error. Thus we have been able to estimate the error in the stress in the “hot-spots” as demonstrated in [3,12,14,15,21]. We have also shown in [3] that in order to obtain reliable estimates of the pollution error, it is essential to employ error indicators which are robust in the interior of the mesh, at the boundary and in the vicinity of the corners.

In [1,2,4,9,18] we developed a computer-based approach for checking the quality of the error indicators for the class of meshes and materials employed in practice. The quality of the error indicators is often measured by *effectivity index*, which cannot be computed in practice. However, it is possible to determine the range of the variation of the effectivity index for a class of meshes, materials and types of problems. We introduced the concept of *robustness-index* which is a measure of the range of the effectivity index of an error indicator for a class of meshes, materials and types of problems and we have given a practical method for computing the robustness-index for any error indicator which maybe also given in the form of a “black-box” in a commercial finite element program. Based on this approach we could check the quality of several error estimators which are used in practice and we have also suggested modifications for the existing estimators to obtain better performance. We concluded that the estimator based on least-squares fit of the stresses (also called *ZZ-SPR* estimator) is the most robust estimator for meshes of triangles and quadrilaterals with straight edges. We have also concluded that the commonly used *explicit estimator* can lead severe overestimation of the energy norm of the error for particular types of meshes of triangles.

In [14,15,21] we gave a new approach for the *a-posteriori* estimation of the error in the quantity of interest and we also gave algorithms for the adaptive mesh-refinement to obtain the stress-intensity factors, for a body with several cracks and reentrant corners, within the prescribed tolerance. The *a-posteriori* estimation of the error in the quantity of interest is based on the construction of an auxiliary solution which is computed by using an extra right-hand side in the global solution. We gave several examples to illustrate that the adaptive mesh-refinement based on controlling the global energy norm of the error does not guarantee the prescribed tolerance for the stress-intensity factors. The algorithms developed in [6,14,15,21] lead to meshes with fewer degrees of freedom than those obtained from the ones based on controlling the global energy norm of the error and moreover, we can guarantee the accuracy of the stress-intensity factors for all the required cracks in the domain.

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**II. Journal Publications**

1. Babuška, I., Strouboulis, T., Upadhyay, C.S., Gangaraj, S.K., and Copps, K., "Validation of A-Posteriori Error Estimators by Numerical Approach", *Intl. J. Numer. Methods Engrg.*, Vol. 37, pp. 1073-1123, 1994.
2. Babuška, I., Strouboulis, T., and Upadhyay, C.S., "A Model Study of Quality of A-Posteriori Error Estimators for Linear Elliptic Problems", *Comput. Meths. Appl. Mech. Engrg.*, Vol. 114, pp. 307-378, 1994.
3. Babuška, I., Strouboulis, T., Mathur, A., and Upadhyay, C.S., "Pollution Error in the  $h$ -Version of the Finite Element Method and the Local Quality of A-Posteriori Error Estimators", *Finite Elements in Analysis and Design*, Vol. 17, pp. 273-321, 1994.
4. Babuška, I., Strouboulis, T., Upadhyay, and C.S., Gangaraj, S.K., "A Model Study of Element Residual Estimators for Linear Elliptic Problems: The Quality of the Estimators in the Interior of Meshes of Triangles and Quadrilaterals", *Computers & Structures*, Vol. 57, pp. 1009-1028, 1995.
5. Babuška, I., Strouboulis, T., Upadhyay, C.S., and Gangaraj, S.K., "Computer-Based Proof of the Existence of Superconvergence Points in the Finite Element Method. Superconvergence of the Derivatives in Finite Element Solutions of Laplace, Poisson's and Elasticity Equation", *Numerical Methods for PDEs*, Vol. 12, pp. 273-321, 1994.
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8. Babuška, I., Strouboulis, T., and Gangaraj, S.K., "A posteriori estimation of the error in the recovered derivatives of the finite element solution", *Comput. Methods Appl. Mech. Engrg.*, to appear (1997).
9. Babuška, I., Strouboulis, T., and Upadhyay, C.S., "A model study of the quality of a-posteriori error estimators for linear elliptic problems. Error estimation at the boundary of patchwise uniform grids of triangles", *Internat. J. Numer. Methods Engrg.*, Vol. 40, 1997.
10. Babuška, I., Strouboulis, T., and Copps, K., " $hp$  Optimization of Finite Element Approximations: Analysis of the Optimal Mesh Sequences in One Dimension", *Comput. Methods Appl. Mech. Engrg.*, Vol. 150, pp. 89-114, 1997.
11. Babuška, I., Ihlenburg, F., Strouboulis, T., and Gangaraj, S.K., "A-posteriori error estimation for finite element solutions of Helmholtz' equation - Part I: The quality of the local error indicators and estimators", *Internat. J. Numer. Methods Engrg.*, Vol. 40, pp. 3443-3461, 1997.
12. Babuška, I., Strouboulis, T., and Gangaraj, S.K., "Practical aspects of a-posteriori estimation and adaptive control of the pollution error for reliable finite element analysis", *Computers & Structures*, Vol. 60, May 1998.

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**II. Journal Publications(continued)**

13. Krizek, M. and Strouboulis, T., "How to generate local refinements of unstructured tetrahedral meshes satisfying a regularity ball condition", *Numerical methods for PDES*, Vol. 13, pp. 201-210, 1997.
14. Babuška, I., Strouboulis, T., Datta, D.K., Copps, K., and Gangaraj, S.K., "A-posteriori estimation and adaptive control of the error in the solution quantities of engineering interest – Part I", (in preparation).
15. Babuška, I., Strouboulis, T., Datta, D.K., Copps, K., and Gangaraj, S.K., "A-posteriori estimation and adaptive control of the error in the solution quantities of engineering interest – Part II", (in preparation).
16. Babuška, I., Strouboulis, T., Gangaraj, S.K., Datta, D.K. and Copps K., "A-posteriori estimation of the error in the error estimate", (in preparation)
17. Strouboulis, T., Babuška, I., and Copps, K., "A-posteriori error estimation of the error in the quantity of interest for the generalized finite element method", (in preparation).
18. Upadhyay, C.S., "Computer-based analysis of error estimation and superconvergence in finite element computation", Ph.D. Dissertation, Texas A&M University, College Station, TX, 283 p. 1997.
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20. Babuška, I., Strouboulis, T., Gangaraj, S.K., Datta, D.K. and Copps K., "A-posteriori estimation of the error in the error estimate", *Proc. of Recent Advances in Adaptive Computational Methods in Mechanics*, Cachan, France, Elsevier (1997).
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**III. Scientific Personnel**

1. Theofanis Strouboulis, Principal Investigator, Associate Professor, Aerospace Engineering, Texas A&M University.
2. Kevin Copps, Ph.D student,
3. Srihari Gangaraj, Ph.D student,
4. Dibyendu Datta, Ph.D student, awarded Master of Science in December 1997.
5. Lin Zhang, Ph.D student,
6. Chandra Upadhyay, awarded Ph.D in May 1997.